The resultant applied force is calculated by integrating the normal stress component under the punch for $-a \le x \le a$, and for $y = -h$ and $-h \le x \le h$. The dimensions of the body used in the analysis are shown in Fig. 6 for the flat punch. Under the punch, the load is found as $P = -180,938$ N/mm; at 'infinity', $P = -180,894$ N/mm, a difference of 0.02%. Using the latter value, the contact length calculated by eq. (13) yields $a/R = 0.141$. Comparison of the normal stress under the punch as obtained by means of the FD method and eq. (14) is shown in Fig. 13; excellent agreement is observed.

5.2 Friction

In this section, friction is assumed between the punch and the substrate. Displacement steps are imposed, as well as iterations to obtain convergence within each displacement step. It was pointed out that in this study displacement was applied to the rigid punch rather than a load. To carry out this process, the FD code was modified. The process is explained. Reference is made to the flow chart in Fig. 14 and Table 6. The displacement steps are denoted by k, whereas, iterations by i. Along the surface of the half-space in the transformed domain $\xi\eta$, either free or slip conditions are assumed. With free conditions, tractions are assumed zero; for slip conditions, eqs. (1) and (2) are imposed. The total displacement Δ is divided by the total number of displacement steps N; so that, $\delta = \Delta/N$. The punch is lowered by an amount δ. The FD solution is obtained. Then, Table 6 is examined. It may be noted that a similar table was presented by [11]. The difference between the tables is in the last two lines in which increments $d(\Delta U_\xi)$ are examined. This change is required for the displacement steps. It may be noted that the displacement and stress fields refer to the numerical domain $\xi\eta$. Returning to the flow chart in Fig. 14, if the assumed conditions at iteration i do not change, then convergence is achieved. If the solution has not converged, Table 6 is used to change the contact zone conditions. Then i is incremented and the FD solution is obtained once again. Iterations continue until convergence is attained. If $k < N$, then k is incremented and the procedure begins again. If $k = N$, the process ends and a solution has been obtained.

It may be noted that the condition in eq. (5) ensures a constant tangential displacement at each point in the stick zone for progressively increasing punch displacements even as the boundary of the contact area changes. The term "frozen in strain" was used by [25] to describe this phenomenon. While a specific behavior of the the frozen in strain is not assumed in this investigation, [25] assumed it to be of the form

$$\frac{\partial U}{\partial x}(y=0) = C_0\,|x|\ , \quad -b \le x \le b\ . \tag{15}$$

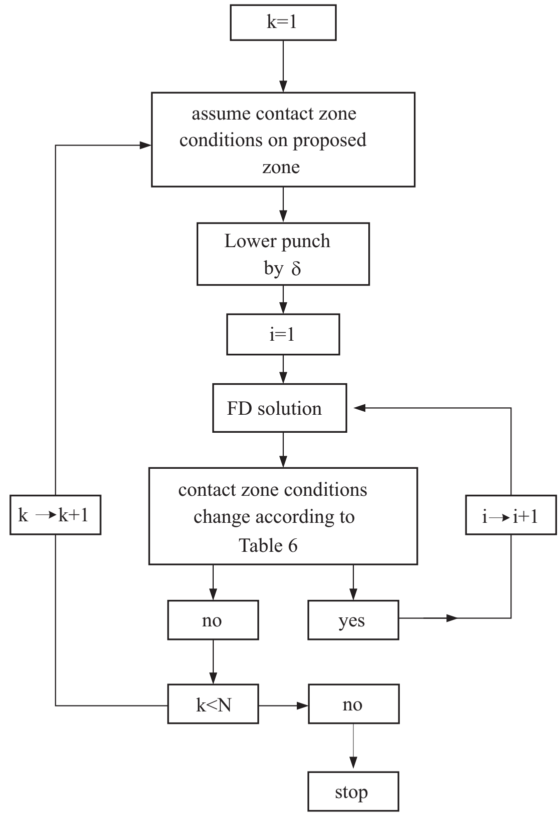

Fig. 14 Flow chart describing displacement steps and iterations.

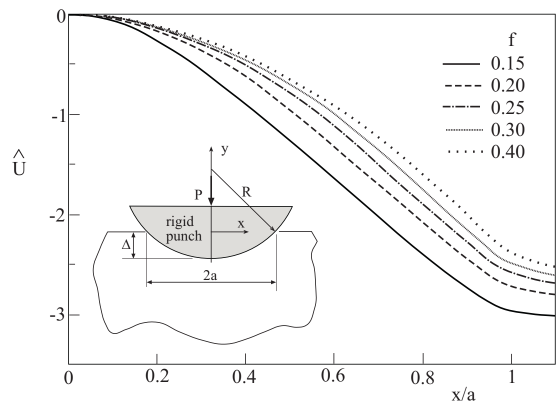

Fig. 15 The tangential displacement along the contact zone for different friction coefficients.

The constant C_0 is a non-positive constant. The linear dependence on x in eq. (15) ensures a constant strain at each point of the stick zone for progressively increasing punch displacements Δ. The value of C_0 is unknown and has to be determined as part of the solution. It is chosen so that the derivative of the displacement $\partial U/\partial x$ at the boundary between the stick and slip zones is continuous. Use of eq. (4) for frictional contact is equivalent to application of the limit case $C_0 = 0$. This is the maximum allowable value of C_0.